THINKING SKILLS **9694/33**

Paper 3 Problem Analysis and Solution **October/November 2017**

2 hours

Additional Materials: Electronic Calculator

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Show your working. Marks may be awarded for correct steps towards a solution, even if the final answer is not correct. Marks may be lost if working needed to support an answer is not shown.

Calculators should be used where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

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1 Fred earns his living as a taxi-driver, offering lifts between his home town of Honeytrees and the airport, a distance of 30 km. He charges a basic rate of $32 for a single journey. The journey in either direction takes at least 40 minutes and at most 70 minutes, depending on the traffic. Fred takes a minimum break of 10 minutes between each single journey. He works a shift of at most 7 hours each day.

Fred's first journey is always from Honeytrees to the airport and his final journey must arrive in Honeytrees before the end of his shift. There are always enough passengers wishing to travel from Honeytrees to the airport. Often he is able to pick up a customer for the return journey. If not, he drives back after his 10 minute break, without a passenger.

(a) What is the greatest amount of money that Fred is able to take in one shift? [2]

Fred decides to change the price that he charges. For each single journey, there will be a basic charge of $28 for the first 40 minutes, plus $0.60 for every further 3 minutes, or part thereof. (This means that as soon as the timer has moved beyond any multiple of 3 minutes, he adds the next $0.60.)

(b) Assuming that there are always sufficient customers at Honeytrees and at the airport, find the greatest amount of money that Fred is **certain** to take in one shift. [3]

The Transport Agency has announced that there will be roadworks between Honeytrees and the airport in both directions for the foreseeable future. This means that journey times will be increased by 20%.

(c) Find Fred's maximum charge for a single journey. [1]

Fred wants to move into a new house and he needs to increase his income. He decides to work a longer shift of up to 9 hours each day. He still has a 10 minute break between each single journey, but after 4 single journeys he has a longer break of 30 minutes. The roadworks are still in place. There are always sufficient customers at Honeytrees, but not necessarily at the airport.

Last Wednesday, Fred made a number of journeys. Two of these journeys took the maximum length of time and the remaining journeys each took 65 minutes.

(d) (i) How long after the beginning of his shift did Fred complete his last journey? [2]

(ii) Find the difference between the greatest and the least amounts of money that Fred could have taken. [2]

2 Moses has a storage room, in the shape of a cuboid, enclosing a space measuring 220 cm horizontally, 240 cm vertically and 600 cm back.
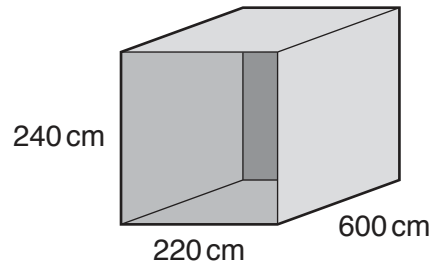
He has 130 identical boxes measuring 50 cm by 60 cm by 80 cm. They can be stacked in any orientation.

Moses stacks boxes into the storage room, all with the same orientation, with the 50 cm edge vertical.

(a) What is the maximum number of boxes he can fit in this way? [2]

(b) What is the largest number of boxes that can be stored if they can be placed in any orientation, but all the boxes must be placed in the same orientation as each other? [2]

Moses' wife, Leillah, claims that there is a way of placing the 130 boxes into the storage room, but not all the boxes would be in the same orientation.

(c) Show that Leillah is correct. [3]

Moses now exchanges his boxes for larger boxes measuring 50 cm by 60 cm by 160 cm, which hold twice as much and are better value. The boxes do not have to be placed in the same orientation as each other.

(d) Moses' daughter, Janet, claims that 63 of these larger boxes can be fitted into the storage room. Is she correct? [3]

3 Tickets for next year's Glastonbourne Music Festival are soon to be released to the general public. They can only be bought by calling the box office, which opens at 8 am on the release date. There are discounted tickets available for the first 100 callers.

Sally is keen to buy discounted tickets, and decides to call before 8 am in the hope of being at the front of the queue when the box office opens. She calls at 6 am precisely and is told by an automated message that she is 560th in the queue. She stays on the line, and one minute later she is told that she is 540th.

(a) Using this information alone, state when Sally should expect to reach the front of the queue. [1]

When Sally reaches the front of the queue, another message informs her that the box office has not opened yet, and that she must try again later. She decides to call again immediately. She assumes that no new people will call the box office, and that everyone else who has already called will also call again immediately after reaching the front of the queue.

(b) If this continues until after the box office opens, at what time will Sally be able to buy her tickets? [1]

However, when Sally calls for the second time, she is told that she is 588th in the queue. She realises that in fact some new people have called the box office. She assumes that these new callers have joined the queue at a constant rate since her first call.

Sally wants to predict, using this new information, when she will be able to buy tickets. She wants to keep her calculations as simple as possible, so she decides to place all her calls precisely at the start of a minute. For example, if she were to arrive at the front of the queue at 7 seconds past 6.49 am, she would place her next call at 6.50 am. If however she were to arrive at 6.49 am exactly, she would place the call immediately, i.e. at 6.49 am.

On this basis, Sally predicts that she will be able to buy tickets at 27 seconds past 8.01 am.

(c) Show how Sally reached her prediction. [3]

Sally wonders if she could avoid waiting unnecessarily in the queue. She uses the information she has so far to work out the best time to call in order to arrive at the front of the queue as soon after 8 am as possible. She will only consider placing the call precisely at the start of a minute.

(d) (i) Calculate at what time she should call, and the precise time she will reach the front of the queue. [2]

(ii) What is the latest time that she could call and still expect to buy discounted tickets? Show at what time she will arrive at the front of the queue. [2]

Sally realises that her assumption that all callers will re-join the queue as soon as they reach the front is unlikely to be realistic. She now assumes that half of the callers will give up and not call again after reaching the front of the queue. So, each time she calls and is told how many people are in the queue, only half of those people will be in the queue the next time she calls.

Because of this extra complexity, Sally decides it will be easier for her to rejoin the queue again each time she reaches the front.

(e) (i) How many new callers does Sally now calculate are joining the queue every minute? [2]

(ii) Will Sally be able to buy discounted tickets? Show what time she should expect to get through to the box office. [4]

[Question 4 begins on the next page]

 [Turn over

4 *Strategy* is a TV quiz show. Four contestants take part in each edition. Throughout the show every question is presented simultaneously to the contestants, together with four possible answers, A, B, C and D, only one of which is correct. Each contestant has an electronic keypad which allows them to select one of the options.

The first part of the show is called *Piggyback*. It consists of five rounds of 10 questions on specific subjects. Every correct answer selected by a contestant adds $5 to their own prize pot.

Before the show begins, the contestants are informed of the five question categories and each of them has to make two strategic decisions:

- One category must be nominated "double". The contestant adds $10 to their prize pot, instead of $5, for every correct answer they select in this round.
- A second category must be nominated "piggyback". The contestant does not take part in this round, but has the same amount of money added to their prize pot for the round as the contestant who adds the most to their prize pot as a result of answering the questions.

None of the contestants knows in advance what categories the others have nominated "double" and "piggyback". This means that a "piggyback" could be worth up to $100 if one or more of the others have nominated the same category "double". Occasionally all four contestants nominate the same category "piggyback", in which case the round becomes void and all the piggybacks are wasted.

The two contestants with the highest prize pots at the end of *Piggyback* progress to the second part of the show, called *Freeze*, taking their prize pots with them.

Freeze consists of 10 general knowledge questions. For each question, the first to select an answer freezes the other out of that answer. The other contestant can then choose to pass or select a different answer. During *Freeze*, every correct answer adds $25 to the contestant's prize pot, but $10 is deducted for every incorrect answer. A contestant who passes because they have been frozen out of the answer they want to select is not penalized, but if neither contestant selects an answer to a question within 10 seconds, $40 is deducted from both prize pots.

The contestant with the higher prize pot at the end of *Freeze* is the show's winner and takes their prize pot through to *Multiplier*, the final part of the show. This consists of 20 general knowledge questions, but with a time limit of only 5 seconds per question. During *Multiplier*, $20 is deducted from the prize pot for an incorrect answer and $40 is deducted if no answer is selected within 5 seconds. The amount of prize money that the winner takes home is their final prize pot multiplied by the number of correct *Multiplier* answers.

Ben, Jodie, Olivia and Toby were today's *Strategy* contestants. The following table details the prize pots they acquired during the *Piggyback* rounds:

	Round 1 History	Round 2 Music	Round 3 Sport	Round 4 Geography	Round 5 Science	Total prize pot
Ben	$40	$90	$45	$80	$40	$295
Jodie	$35	$45	$40	$25	$45	$190
Olivia	$80	$40	$35	$80	$45	$280
Toby	$35	$90	$45	$35	$35	$240

In *Freeze*, Olivia was first to select an answer to all but the last of the 10 general knowledge questions. She appeared to make no effort at all to answer the last question. The performances of Ben and Olivia during *Freeze* are summarized below.

Question	1	2	3	4	5	6	7	8	9	10
Ben	pass	correct	pass	incorrect	pass	pass	pass	pass	correct	pass
Olivia	correct	incorrect	incorrect	correct	incorrect	correct	correct	correct	incorrect	pass

Olivia was today's winner. She went on to answer 12 of the *Multiplier* questions correctly, but answered 7 incorrectly and failed to answer the other one within the 5-second time limit.

(a) What is the largest amount of money that anyone could possibly win on an edition of *Strategy*? [2]

(b) (i) How much was in Ben's prize pot and how much was in Olivia's prize pot after today's **ninth** *Freeze* question? [2]

(ii) Suggest why Olivia did not attempt to answer the last question. [1]

(c) (i) In total, how many questions did Olivia answer correctly today? [2]

(ii) How much did Olivia win? [2]

(iii) What is the maximum extra amount Olivia could have won if she had selected an answer to the last *Freeze* question? [2]

(d) Deduce which categories were each of today's four contestants' "double" and "piggyback" choices. [4]

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